Dudeney's Haberdasher puzzle

Author: Ivan De Winne.
Subject: Mathematics.
Country of creation: Belgium.
Countries of testing: Czech Republic, Portugal and Ireland.

Aims of the GP

Building new mathematical knowledge through problem solving.

Make and test conjectures about geometric properties.

Develop logical arguments to justify conclusions.

Make geometric constructions by using GeoGebra.

Age of the students

13-16

Preparation and teaching time

4-5 lessons

Teaching material

- Paper and a pair of scissors.
- Computer with Internet connection.
- GeoGebra: software for dynamic geometry and algebra.
- Websites concerning Dudeney.
- A data projector.

Lesson plan

Step 1

- Short presentation of the problem: "Make a dissection of an equilateral triangle into four pieces that can be put together to make a perfect square."
- Appetizer activity: applet + puzzle (30 minutes).
- A school logo (see image, as an example) is used to cut a square into 4 pieces and to create an equilateral triangle.

Step 2

- Collecting a few interesting links by using some keywords: Dudeney, Canterbury puzzles, hinged dissections, etc. (1 h).

- Dudeney's Haberdasher puzzle is introduced with a screen capture from the first publication in 1908 in the book "Canterbury puzzles".

Step 3: Simple step by step construction of Dudeney's puzzle with GeoGebra (1h).

Step 4: Calculations of the area of the triangle and the exact length of the side of the square. Students must compare the "constructed" value with the exact length of the side of the square (30 minutes).

Step 5: Comparing simple (approximate) solutions with the exact solutions proposed by Dudeney in 1908 (1 h).

Step 6: Students should be able to make a dynamic applet with GeoGebra to demonstrate the triangle to square transformation! (1h).

Questionnaire

If the length of a side of an equilateral triangle is 4, then the area of the triangle is

6 cm²
8 cm²
12 cm
2*√12 cm²

The side of the square in the figure is 3 cm. The perimeter of the square is

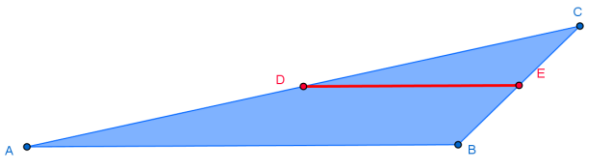
9 cm
3 cm
12 cm
6 cm²

In an equilateral triangle all interior angles are equal to 60°.

Yes
No

In the figure below the longest side of the triangle has the length 10 cm.
The length of the segment DE is:

3 cm
4 cm
5 cm
6 cm



In a triangle ABC all sides are equal and BT perpendicular to AC. Then

AT = TC

AB < BT
AT > AC
AC < BC

Constructing an equilateral triangle with only a compass and a ruler is:
Possible
Impossible

Comparing rational and irrational numbers
$\frac{7}{4} \neq \sqrt{3}$ (correct) $\frac{7}{4} < \sqrt{3}$

$\frac{7}{4} = \sqrt{3}$ $\frac{\sqrt{7}}{2} > \sqrt{3}$

Henry Ernest Dudeney was a famous English mathematician. He published a very popular collection of maths puzzles in a book called
Dudeney's tales.
Canterbury puzzles.
Brainteaser puzzles.
Canterbury tales.

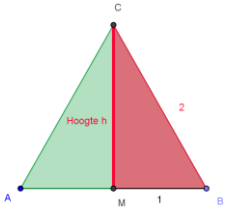
Dudeney's dissection (see figure) is a dissection of

A square into 6 pieces which can be rearranged into an equilateral triangle.
An equilateral triangle to a square, with only four cuts.
An equilateral triangle into 4 pieces which can be rearranged into a square with the same area as the triangle.
A square into 4 pieces which can be rearranged into a hexagon.

Calculate the height h of the equilateral triangle with side 2

$h = \sqrt{5}$ $h = \sqrt{3}$ (correct)

$h^2 = 5$ $h = 3$



Teacher reviews

This GP also addresses various learning goals. For instance, an Irish teacher gave as feedback before implementing this GP that he had "what is called a gentleman's knowledge of GeoGebra but did not really understand its full potential" and therefore he wanted to learn something more himself, which he would then happily pass on to his students who are "very interested in hand to eye exercises worked on graph paper." He quickly realized that "the students

had massive prior knowledge on grids, axes, co-ordinates, quadrants, lines, angles, bisections and various types of triangles and other 2D shapes. This was the obvious progression and implementation on the IWB magnified and illustrated their activities" And he added: "Thanks to this GP and the success it showed with the students, at the moment we have two GeoGebra groups in our school actively engaged in this subject".

This GP was also a success, according to the Czech teacher, as the students, who did not know how to work with GeoGebra, learned quickly and enjoyed working with it. They seemed to particularly enjoy the combination of group work and working with the computers.

The SPICE project

SPICE was a two-year project (December 2009 – November 2011) carried out by **European Schoolnet** (http://europeanschoonet.org) together with **Direção Geral de Inovação e Desenvolvimento Curricular** (http://sitio.dgidc.min-edu.pt/Paginas/default.aspx) from Portugal and **Dum Zahranicnich Sluzeb MSMT** (http://www.dzs.cz/) from the Czech Republic.

The primary objective of the SPICE project was to collect, analyse, validate and share innovative pedagogical practices, particularly those using inquiry-based learning, whilst enhancing pupils' interest in the sciences. SPICE supported this objective by singling out, analysing and validating good practice pedagogies and practices in maths, science and technology (mostly ICT-based) and disseminating them across Europe. SPICE involved 24 teachers from 16 different educational systems (from 15 different countries). This teachers' panel helped the SPICE partners in defining good practices that were then tested in classes by 41 teachers during the school year 2010-2011.

For more information see: **http://spice.eun.org**

Education and Culture DG
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